several times. The body pitch rate manifests more flailing behavior as can be seen from the broken line $\hat{\psi}/10$ in Fig. 3. The reconstructed radome error curve $\hat{r}\cdot H$, as shown by the dots, still follows very closely the boresight error (the solid curve Z). The comparison of the estimated slope and the actual one was very difficult due to the ambiguity in the average and local slope calculations during the hybrid run. By not estimating the cross-plane slope simultaneously, we make the missile roll attitude contribution inseparable.

After imbedding this adaptive estimator into terminal guidance forward simulation program to perform the closed-loop analysis, some encouraging results have been obtained. Depending on the actual radome error and the approaching trajectory, the improvement in the miss distance performance index ranges 20-50%. In Ref. 1, the estimated radome error slope is used in guidance and autopilot commands to provide an optimal pitch rate compensation scheme for a modified proportional navigation system and an optimal controller.[2] Using P_+, the probability of radome error being in the positive and negative regions, to scale the pitch rate compensation for enlarging the radome stability region, has also demonstrated reduced excitation in acceleration commands.

Conclusion

A Kalman filter bank is designed to enhance the dynamic response time for the radome error slope estimate with compensation for the seeker dynamic lag. In calculating the critical weighting coefficients (a posteriori probabilities) a measure-predict-measure technique is used when the semi-Markov statistics of a random starting process are used to make the intermediate predictive step. That is, the resulting estimated radome slope parameter is the statistical average weighted by time-varying, a posteriori hypothesis probability, which is calculated concurrently with the recursive filter scheme by using Bayesian rule. To reduce computation burden, the Kalman filter bank is digitally simulated and designed by tuning the noise processes, including the measurement and plant noise, to allow a one-time calculation of the Kalman filter gain. The simple one-state filter described above can be modified to include a correlation parameter for studying the cross-plane errors. The bank of Kalman filters can be increased to five to enlarge the dynamic range while reducing the system response time simultaneously and to estimate the cross-plane radome error slope simultaneously for three-dimensional engagement. The adaptive radome estimator design is intended to be an "add-on" compensation network that is independent of guidance computer and autopilot design. The objective is to permit relaxation of missile bandwidth requirements by reducing error due to radome at the guidance computer output, thus enhancing missile performance.

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Controllability and Observability of General Linear Lumped-Parameter Systems

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Introduction

THE dynamic behavior of a general linear discrete system can often be represented by the second-order vector differential equation

$$M\ddot{x}(t)+\tilde{C}\dot{x}(t)+\tilde{K}x(t)=0 \tag{1}$$

where M, $\tilde{C}$, and $\tilde{K}$ are the mass, damping, and stiffness matrices. Here, $\tilde{C}$ and $\tilde{K}$ are general asymmetric matrices that include general types of forces (e.g., elastic, nonconservative, dissipative, and gyroscopic).[1] Assuming M to be nonsingular (i.e., det $M\neq 0$), Eq. (1) can be simplified as

$$\ddot{x}(t)+C\dot{x}(t)+Kx(t)=0 \tag{2}$$

where

$$C=M^{-1}\tilde{C} \tag{3a}$$

$$K=M^{-1}\tilde{K} \tag{3b}$$

The presence of control forces modifies the equation of motion as

$$\ddot{x}+C\dot{x}+Kx=f \tag{4}$$

where

$$f=Bu \tag{5}$$

is the $n_a\times 1$ control vector and B is the influence matrix for the n_a control actuators. Additionally, if n_0 sensors are used for separate or mixed measurements of displacement and velocity, then the n_0 sensor outputs can be represented by

$$y=Px+R\dot{x} \tag{6}$$

where y is the output vector and the matrices P and R represent the displacement and velocity influence coefficients, respectively.

It is well known that, based on a first-order formulation of the system, it is possible to mathematically define the controllability and observability of the system as[2]

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$$V=[B_1,AB_1,\ldots,A^{2n-1}B_1],\ \text{rank}\ (V)=2n \tag{7}$$

$$W=[C^T,A^TC^T,\ldots,(A^T)^{2n-1}C^T],\ \text{rank}\ (W)=2n \tag{8}$$

where

$$A=\begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix},\quad B_1=\begin{bmatrix} 0 \\ B \end{bmatrix},\ C=[P\ R]$$

Although these rank conditions are conceptually simple, they may create computational difficulties for systems with many degrees of freedom.

In the past few years, several studies have been devoted to developing more efficient conditions by exploiting the special form of the systems discussed here. In a study by Hughes and Skelton,[3] a decoupled form of Eq. (1), i.e.,

$$\ddot{q}+\hat{K}q=\hat{B}u \tag{9a}$$

$$y=\hat{P}q+\hat{R}\dot{q} \tag{9b}$$

where q is the vector of decoupled coordinates, has been used to find conditions on the controllability and observability of symmetric conservative systems. Partitioning the vector $\hat{B}$ according to the multiplicities of eigenvalues of the matrix $\hat{K}$, denoted by ω_i, as

$$\hat{B}=\begin{array}{c} \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \\ \cdot \\ \cdot \\ \cdot \\ \hat{B}_\alpha \end{bmatrix} \end{array}\begin{array}{l} n_1 \text{ rows} \\ n_2 \text{ rows} \\ \\ \\ \\ n_\alpha \text{ rows} \end{array} \qquad (n_1+n_2+\ldots+n_\alpha=n)$$

shows that the system of Eq. (1) is controllable if and only if

$$\text{rank}(\hat{B}_i)=n_i,\ i=1,2,\ldots,\alpha$$

In addition, the results in Ref. 3 suggest that Eq. (1) is observable if and only if

$$\hat{Q}_i=\begin{bmatrix} \hat{P}_i & \hat{R}_i \\ \omega_i\hat{R}_i & \hat{P}_i \end{bmatrix},\ \text{rank}\ (\hat{Q}_i)=2n_i,\ i=1,2,\ldots,\alpha$$

where the matrices $\hat{P}_i$ and $\hat{R}_i$ are partitions of $\hat{P}$ and $\hat{R}$, respectively, according to the multiplicities of ω_i, just as $\hat{B}$ was partitioned. For special cases of Eq. (2), several other controllability and observability results are given that, in addition to simplifying the controllability and observability conditions, provide an estimate of the number of actuators and sensors required to control a system.

In another study, Juang and Balas[4] considered conservative gyroscopic systems, such as occur in large spinning spacecrafts. Using a decoupled first-order form of the system, this study provides a simplified set of controllability and observability conditions that can also be used to determine the minimum number of actuators and sensors required for active modal control of flexible large spinning spacecrafts. The results in Ref. 4 are applied to the lumped-parameter model of a radio astronomy Explorer (RAE/B) satellite to illustrate that only 2 collocated actuators and sensors are needed to control 30 modes of the model.

In a more recent study by Inman and Hsieh,[5] active control of pseudoconservative systems is discussed. In this study, it is suggested that the controllability and observability conditions developed in Ref. 3 can be extended to this class of systems.

The main object of the present study is to extend the results on controllability and observability, as proposed in Refs. 3-5, to a broader class of systems. The model of a lumped-parameter system with general types of forces, as shown in Eq. (4), is considered for this purpose. Next, easily verified controllability and observability conditions are developed for such systems. It is shown that these conditions may be more computationally attractive for high-order systems, since they replace the rank conditions [Eqs. (7) and (8)] with a number of lower-order conditions. In addition, they provide a mode-by-mode analysis of the controllability and observability and give an estimate of the minimum number of sensors and actuators required to accomplish control.

Results

The general system described in Eq. (4) can alternatively be written as

$$\ddot{x}+D\dot{x}+G\dot{x}+Kx=Bu \tag{10}$$

where the terms $D\dot{x}$ and $G\dot{x}$ represent the damping and gyroscopic forces, respectively. For the moment, consider the undamped case characterized by $D\dot{x}=0$. This allows Eq. (10) to be presented in a first-order form as

$$\begin{bmatrix} K & 0 \\ 0 & I \end{bmatrix}\dot{q}=\begin{bmatrix} 0 & K \\ -K & -G \end{bmatrix}q+\begin{bmatrix} 0 \\ B \end{bmatrix}u \tag{11}$$

where

$$q=\begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{12}$$

is the state vector. Similarly, the output vector in Eq. (6) can be written as

$$y=Cq \tag{13}$$

where

$$C=[P\ R] \tag{14}$$

Assuming the matrix K to be symmetrizable with positive eigenvalues, i.e.,

$$K=S_1S_2,\quad S_1=S_1^T>0,\quad S_2=S_2^T>0 \tag{15}$$

further simplifies Eqs. (11) and (13), as shown below

$$\dot{e}=\begin{bmatrix} 0 & K^{1/2} \\ -K^{1/2} & -G \end{bmatrix}e+\begin{bmatrix} 0 \\ B \end{bmatrix}u, \tag{16a}$$

$$y=[K^{-1/2}P\ R]e \tag{16b}$$

where

$$e=\begin{bmatrix} K^{1/2}x \\ \dot{x} \end{bmatrix} \tag{17}$$

Here, the matrix $K^{1/2}$, defined as the square root of the symmetrizable matrix K, can be presented in terms of the factors S_1 and S_2 as

$$K^{1/2}=S_1^{1/2}(S_1^{1/2}S_2S_1^{1/2})^{1/2}S_1^{-1/2} \tag{18}$$

The validity of $K^{1/2}$ can be verified[6] by simply considering the matrix K as presented in Eq. (15) and testing $K=K^{1/2}K^{1/2}$.

It is known that there exists an orthogonal transformation ϕ which transforms the skew symmetric matrix

$$\bar{A}=\begin{bmatrix} 0 & K^{1/2} \\ -K^{1/2} & -G \end{bmatrix} \tag{19}$$

into a real Jordan (Murnaghan) form

$$A=\phi^T\bar{A}\phi=\mathrm{Diag}[A_1,A_2,\ldots,A_\alpha] \tag{20a}$$

The diagonal entries A_j, $j=1,2,\ldots,\alpha$ are the direct sum of 2×2 matrices

$$\begin{bmatrix} 0 & \omega_j \\ -\omega_j & 0 \end{bmatrix} \tag{20b}$$

with dimensions of A_j equal to $2n_j$, where n_j is the multiplicity of the mode frequency ω_j.[1] Substituting the decoupled coordinates

$$z=\phi^T e \tag{21}$$

into Eqs. (16) and premultiplying Eq. (16a) by ϕ^T results in

$$\dot{z}=Az+\bar{B}u \tag{22a}$$

$$y=\bar{C}z \tag{22b}$$

where

$$\bar{B}=\phi^T B \tag{23a}$$

$$\bar{C}=C\phi \tag{23b}$$

and A is as defined in Eq. (20a). Partitioning the matrices $\bar{B}$ and $\bar{C}$ according to the blocks A_j as

$$\bar{B}=\begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_\alpha \end{bmatrix} \tag{24}$$

where the rows (columns) of $B_j(C_j)$ are equal to $2n_j$, yields the following theorems.

Theorem 1. The system of Eq. (10) is controllable if and only if

1) $n_c\geq\max(n_1,n_2,\ldots,n_\alpha)$.
2) The $2n_j\times 2n_jn_c$ matrix

$$[B_j,A_jB_j] \tag{25}$$

has rank $2n_j$.

Proof: The blocks A_j do not share any frequencies, since A_j is the direct sum of 2×2 matrices, shown in Eq. (20), whose squares produce scalar multipliers of the identity matrix, i.e.,

$$A_j^2=\omega_j^2 I \tag{26}$$

Therefore, according to the theorem 9.3 in Ref. 7, the system of Eq. (22) is controllable if and only if $n_c\geq n_j$ and (A_j,B_j) is controllable, i.e.,

$$\mathrm{rank}\,[B_j,A_jB_j,\ldots,A_j^{2nj-1}B_j]=2n_j$$

But, due to the special form of Eq. (26), this can be reduced to

$$\mathrm{rank}\,[B_j,A_jB_j]=2n_j$$

Thus, Eq. (10) is controllable if and only if conditions 1 and 2 are satisfied.

Theorem 2. The system of Eq. (10) is observable if and only if

3) $n_0\geq\max(n_1,n_2,\ldots,n_\alpha)$.
4) The $2n_j\times 2n_jn_0$ matrix

$$\begin{bmatrix} C_j \\ C_jA_j \end{bmatrix} \tag{27}$$

is of rank $2n_j$.

Proof: The proof follows directly from what was stated for theorem 1.

Effects of Energy Dissipation

Although the preceding theorems were developed based on an undamped model, they remain valid when the effects of damping forces, as appearing in Eq. (10), are present in the system. The addition of the term $D\dot{x}$ provides a new coupling mechanism between modes that may cause the uncontrollable and unobservable modes with $D=0$ to become observable and controllable when $D\neq 0$. Because of this coupling effect, fewer actuators and sensors may be needed to establish controllability and observability for a sufficiently damped system. However, when the damping mechanism is weak, it is recommended that controllability and observability be established based on the undamped model (i.e., $D=0$) and that the damping forces be regarded as a secondary influence on these characteristics.[3]

The effects of damping on control have been addressed in many other studies, such as Refs. 3, 8 (theorem 1.3), and 9, where it is also concluded that such forces can be ignored safely in most cases. Consequently, theorems 1 and 2 can effectively be applied to general lumped-parameter systems as modeled in Eq. (4).

Observations

The tests for the controllability and observability of lumped-parameter dynamic systems, as viewed by different authors in Refs. 3-5, have been extended to include a broader class of systems in which general forces are present. Although the approach presented here requires several transformations to be applied to the system, it establishes a mode-by-mode analysis, rather than a total system analysis, of the system and replaces a single rank condition with a number of lower-order conditions. The advantage of the latter is most appreciated when dealing with the analysis of high-order systems, such as those resulting from large space structures.

An interpretation of this study is that only a few actuators and sensors may be needed to control a large number of system modes. The minimum number of sensors and actuators can be determined by direct examination of the conditions 1 and 3, respectively. For instance, these conditions indicate that a system with distinct frequencies may be controlled completely by a single, properly located actuator and sensor, which is similar to the observation made on the control of symmetric systems in Refs. 3 and 4.

The presented approach may be programmed for actuator and sensor placement on a structure. To accomplish this, a numerical algorithm similar to that presented in Ref. 10 can be used to transform the physical coordinates to modal coordinates [i.e., decouple the system as shown in Eqs . (22)]. Next, the minimum number of control devices are determined from conditions 1 and 3 and are located on the structure. If the rank conditions 2 and 4 are satisfied, then the locations are acceptable. Otherwise, through an iterative routine, new locations are chosen and the rank conditions 2 and 4 are checked until satisfactory results are obtained. It is worthwhile to note that what is being suggested here is not meant to replace the designer's intuition about the location of sensors and actuators. Instead, it is intended to enhance that intuition when dealing with the difficult issues of active control.

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Book Announcements

KAILATH, T., Stanford University, ***Lectures on Wiener and Kalman Filtering,*** Springer-Verlag, New York, 1981, 187 pages. $16.90.

Purpose: This is a revised edition of the book, ***Lectures on Linear Least Squares Estimation,*** published in 1975. Several errors have been corrected; a few minor changes have been made, and a survey paper on disctrete-time estimation has been added as an appendix. The book is intended as a text for a first course on estimation theory.

Contents: Introduction. Least-square estimates, basic properties. Wiener filters. Generalizations of Wiener filtering. Discrete-time recursive estimation and the Kalman filter. Continuous-time Kalman filters. Relations to Wiener filters. Recursive Wiener filters. Fast algorithms for constant parameter models. Some related problems. Appendices.

KRISHNAN, V., Indian Institute of Science and University of Lowell, ***Nonlinear Filtering and Smoothing: An Introduction to Martingales, Stochastic Integrals and Estimation.*** John Wiley and Sons, New York, 1984, 314 pages. $34.95.

Purpose: This book is the outcome of a course on martingales and estimation theory given to engineering graduate students with a basic knowledge of probability theory. The emphasis is on a concise physical understanding of the principles of martingales, stochastic integrals, and estimation theory from an application point of view.

Contents: Basic concepts of probability theory. Stochastic processes. Martingale processes. Classes of martingales and related processes. White noise and white noise integrals. Stochastic integrals and stochastic differential equations. Stochastic differential equations. Optimal nonlinear filtering. Optimal linear nonstationary filtering (Kalman-Bucy filter). Application of nonlinear filtering to fault detection problems. Optimal smoothing. References. Index.

BLOMBERG, H., Helsinki University of Technology, and **YLINEN, R.,** Technical Research Center of Finland, ***Algebraic Theory for Multivariable Linear Systems.*** Academic Press, San Diego, 1983, 360 pages. $48.00.

Purpose: This book is based on research performed over a number of years on the analysis of linear systems with nonzero initial conditions. The methodology is referred to as polynomial system theory or generalized transfer functions.

Contents: Introduction. Systems and system descriptions. Interconnections of systems. Generation of differential systems. The C[p]-module signal space. Differential input-output relations (generators). Analysis and synthesis problems. The projection method. Interconnections of differential systems. Generation of difference systems. The module structure. Difference input-output relations (generators). Analysis and synthesis problems. The vector space structure, the projection method. Appendices. References. Index.

BUTKOVSKIY, A.G., Academy of Sciences of the USSR, ***Structural Theory of Distributed Systems,*** translated by L.W. Longdon. Ellis Horwood Limited, Chichester, England, 1983, 314 pages. $84.95.

Purpose: This book is concerned with the development of block diagram techniques for distributed parameter systems, that is, systems described by partial differential equations. It is directed toward specialists in science and engineering.

Contents: General theory of structural diagrams for distributed parameter systems. The structural representation of some applied problems. The structural representation of problems on elastic structures. The quenching of oscillations in distributed lines. Controllability, finite control, observability, synthesis of linear distributed systems. Appendix. Bibliography. Index.